A short table of trigonometric identities

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Pythagorean Identity:

$$\sin^2\alpha+\cos^2\alpha=1$$

Sum-Difference Formulas:

$$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\cos\alpha\sin\beta$$

$$\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta$$

Even-Odd Identities:

$$\sin(-\alpha)=-\sin\alpha$$

$$\cos(-\alpha)=\cos\alpha$$

Phase Shift Identities:

$$\sin(\alpha\pm360^\circ)=\sin\alpha$$

$$\cos(\alpha\pm360^\circ)=\cos\alpha$$

$$\sin(\alpha\pm180^\circ)=-\sin\alpha$$

$$\cos(\alpha\pm180^\circ)=-\cos\alpha$$

$$\sin(\alpha\pm90^\circ)=\pm\cos\alpha$$

$$\cos(\alpha\pm90^\circ)=\mp\sin\alpha$$

Product-to-Sum Formulas:

$$\cos\alpha\cos\beta=\frac{1}{2}\cos(\alpha+\beta)+\frac{1}{2}\cos(\alpha-\beta)$$

$$\sin\alpha\sin\beta=\frac{1}{2}\cos(\alpha-\beta)-\frac{1}{2}\cos(\alpha+\beta)$$

$$\sin\alpha\cos\beta=\frac{1}{2}\sin(\alpha+\beta)+\frac{1}{2}\sin(\alpha-\beta)$$

$$\cos\alpha\sin\beta=\frac{1}{2}\sin(\alpha+\beta)-\frac{1}{2}\sin(\alpha-\beta)$$

Double Angle Formulas:

$$\sin(2\alpha)=2\sin\alpha\cos\alpha$$

$$\cos(2\alpha)=\cos^2\alpha-\sin^2\alpha=2\cos^2\alpha-1=1-2\sin^2\alpha$$

Half Angle Formulas:

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

Rectangular-to-Polar and Polar-to-Rectangular:

$$A\cos\alpha + B\sin\alpha = C\cos(\alpha+\theta) = C\cos(\alpha-\varphi)$$

where:

$$C = \sqrt{A^2 + B^2}$$

$$\theta = \tan^{-1}\frac{-B}{A}$$

$$\varphi = \tan^{-1}\frac{B}{A}$$

$$A = C\cos\theta = C\cos\varphi$$

$$B = -C\sin\theta = C\sin\varphi$$

Complex Numbers:

$$\sin\alpha = \frac{e^{j\alpha} - e^{-j\alpha}}{2j}$$

$$\cos\alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$$

$$e^{\pm j\alpha} = \cos\alpha \pm j\sin\alpha$$